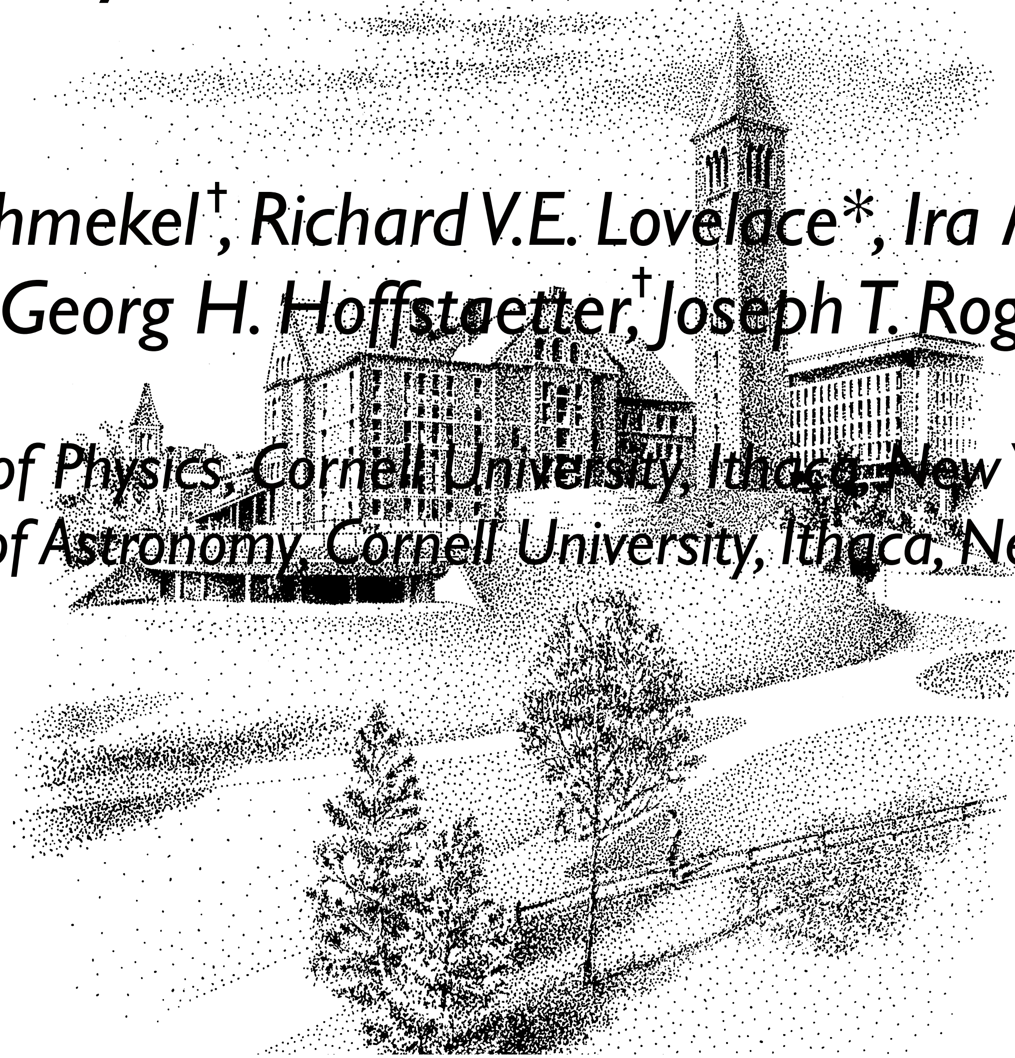
Progress in Understanding the Coherent Synchrotron Radiation Instability



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Problems in Astrophysics:

Coherent Synchrotron Radiation, Two-Stream Instability, Cyclotron Maser Instability etc.

Problems in Beam Physics:

Coherent Synchrotron Radiation, Two-Stream Instability, Cyclotron Maser Instability etc.

Consider the following equilibrium distribution

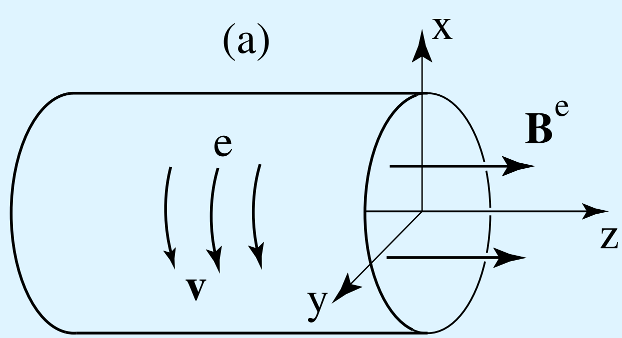
Rigid rotor equilibrium

$$f^0(H, P_\theta) = N_0 \delta(P_\theta - P_0) \exp\left(-\frac{H - \gamma_0 m_e c^2 + e\bar{\phi}_0}{\sigma}\right)$$

cylindrical symmetry

small thermal energy spread

external magnetic field in the z-direction



Linear Stability Analysis

- For which parameters does an arbitrary initial perturbation grow?
- What is the growth rate?
- How big is the saturation amplitude and the radiated power? (Linear analysis is not sufficient to answer these questions.)

Previous Work

- Goldreich and Keeley 1971 (1 dimensional)
- Heifets and Stupakov 2002 (1 dimensional with particles moving on different orbits)
- Byrd 2003 (1 dimensional with beampipe)
- Uhm, Davidson et al. 1985 (2 dimensional, no betatron oscillations, beam pipe, different equilibrium etc.)

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COHERENT SYNCHROTRON RADIATION*

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ABSTRACT

A simple model consisting of a distribution of charges constrained to move on a ring is the basis of an investigation of coherent synchrotron radiation. The radiation produced as a result of a nonrandom particle distribution on the ring is examined from the viewpoint of the interaction of individual particles with the total electric field of the system. A linear stability analysis shows that, under reasonable conditions, a uniform distribution of particles is unstable to clumping. The model is applied to pulsars, in which the high brightness temperatures suggest that a cooperative emission mechanism is responsible for the radiofrequency radiation. The application to circular accelerators and storage rings is discussed briefly.

I. INTRODUCTION

The main purpose of this paper is to show in detail how the power radiated by a system of charges may be understood in terms of the interactions of individual particles through the electromagnetic field. The physical situation which will be discussed is a tion (14) is insensitive to a spread in direction $\Delta\alpha$ if $\Delta\alpha \lesssim n^{-1/3}$; this condition is easily satisfied for tubes of the thickness considered above.

c) Circular Accelerators and Storage Rings

A one-dimensional continuity equation has been used in the preceding analysis. This will not generally be a good approximation for the motions of particles in accelerators or storage rings. Thus the analysis of the instability is not applicable in general. Even for cases where the approximation is not too bad, the conditions for instability, and for negligible interference caused by the energy spread in the beam, are not easily satisfied. Under most conditions the growth rate $s/\omega_0 \ll 1$. In addition, the presence of metal surfaces near the beam would probably tend to damp the instability. Thus it seems unlikely that the instability will be important for high-energy particle machines.

VII. CONCLUSIONS

No very strong conclusions can be drawn in the absense of a detailed model for the region near the magnetic poles of the neutron star. However, it has been shown that an

Vlasov-Maxwell Equations

To first order in the perturbation amplitude the Vlasov equation is

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}+\frac{d\mathbf{p}}{dt}\cdot\frac{\partial}{\partial \mathbf{p}}\right)\delta f=\frac{D\delta f}{Dt}=e(\delta\mathbf{E}+\mathbf{v}\times\delta\mathbf{B})\cdot\frac{\partial f^0}{\partial \mathbf{p}}$$

We also need the Maxwell equations and

$$v_\perp=\frac{p_\perp}{m_e} \qquad v_\theta=\frac{p_\theta}{m_e}\left(1+\frac{p_r^2+p_\theta^2}{m_e^2c^2}\right)^{-1/2}$$

$$\delta\rho=\int d^3p\,\delta f \qquad \delta\mathbf{j}=\int d^3p\,\mathbf{v}\delta f$$

All perturbed quantities have the dependence $e^{im\phi+ik_zz-i\omega t}$

Finally, ...

... the following eigenvalue equation can be obtained

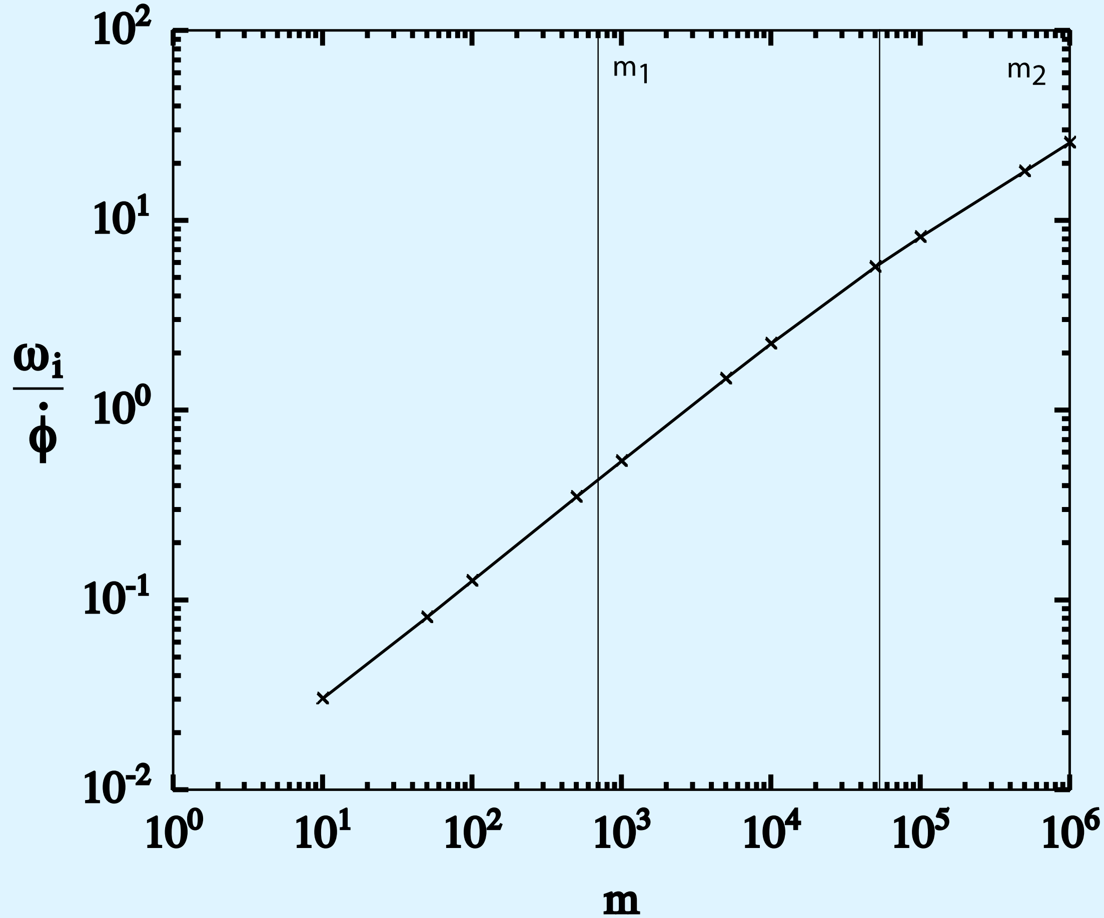
$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}-\frac{m^2}{r^2}+\omega^2\right)\delta E_\phi(r)=4\pi e^2\delta E_\phi(r)n(r)\left(\frac{m}{r}-\omega r\dot{\phi}\right)\frac{m\dot{\phi}^2}{H}\frac{r}{(\omega-m\dot{\phi})^2}$$

which can be rewritten in terms of the following three dimensionless quantities

$$\gamma=\frac{H_0}{m_e} \qquad v_{th}=\sqrt{\frac{\sigma}{H_0}} \qquad \zeta=\frac{4\pi e n_0 v_{\phi 0} r_0 v_{th}\sqrt{\pi/2}}{B_z^e}$$

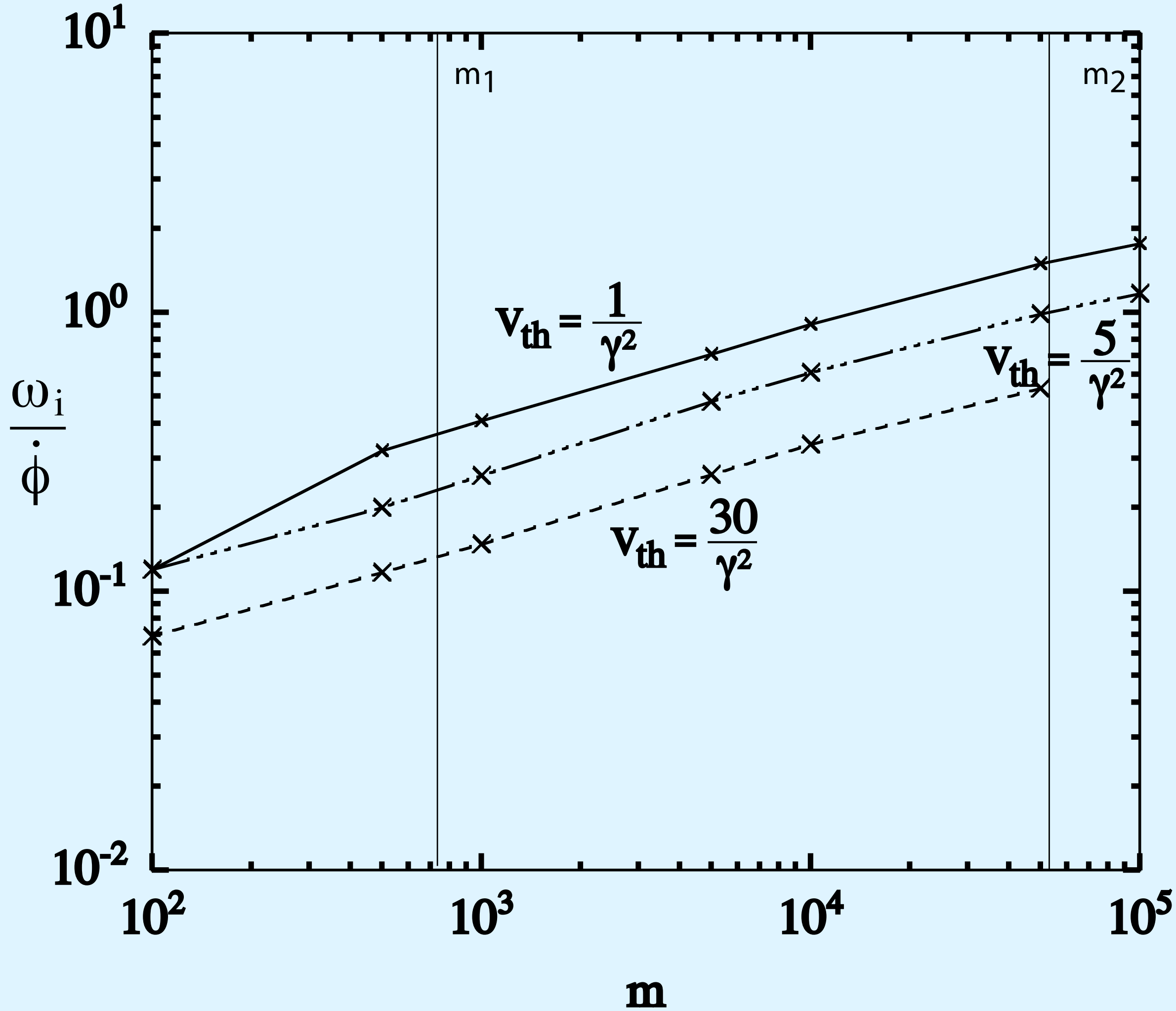
Boundary conditions:
The perturbation in the electric field should be regular at the origin and approach zero for r→∞

Results in the thin limit



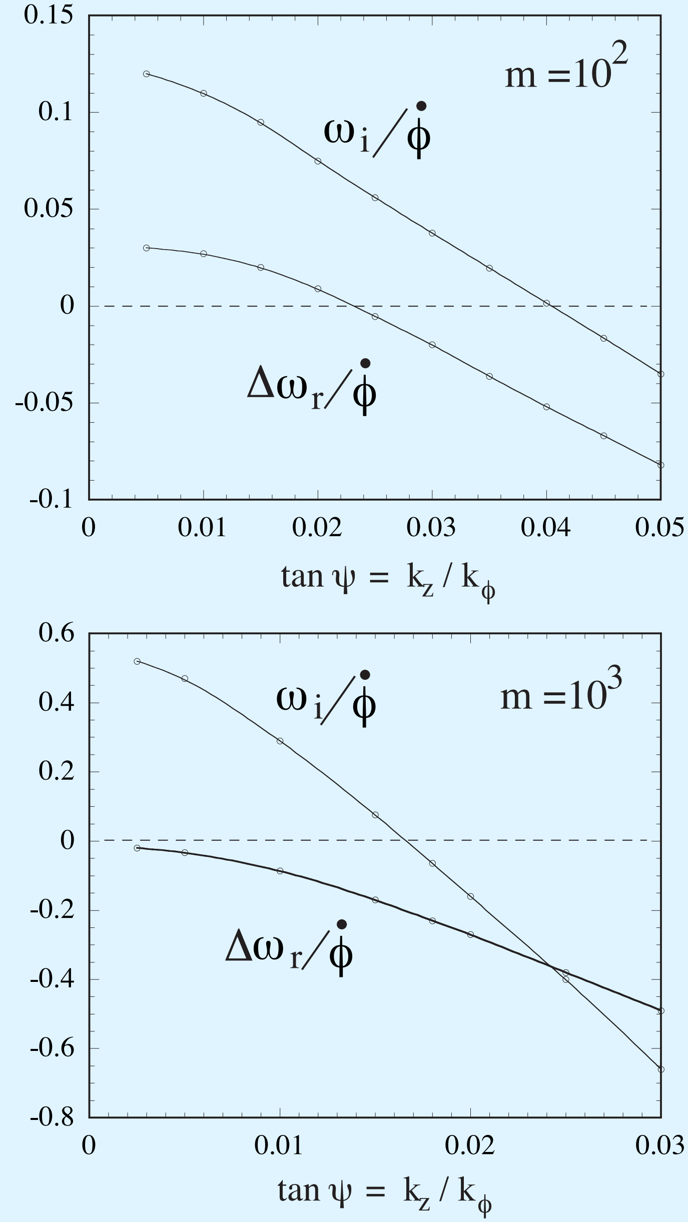
Growth rate for γ=30 and ζ=0.02

Results in the thin limit including betatron oscillations



Growth rate for γ=30 and ζ=0.02 including betatron oscillations

Inclusion of axial modes



Brightness Temperature

In linear perturbation theory the exponential growth lasts forever. We need the saturation amplitude in order to compute the brightness temperature, though.
For a relativistic particle on a circular orbit (Lawson 1988)

$$\delta P_\theta = m_{e*} r_0^2 \delta\dot{\phi} \qquad m_{e*} = \frac{-m_e\gamma^3}{\gamma^2-1} \approx -m_e\gamma$$

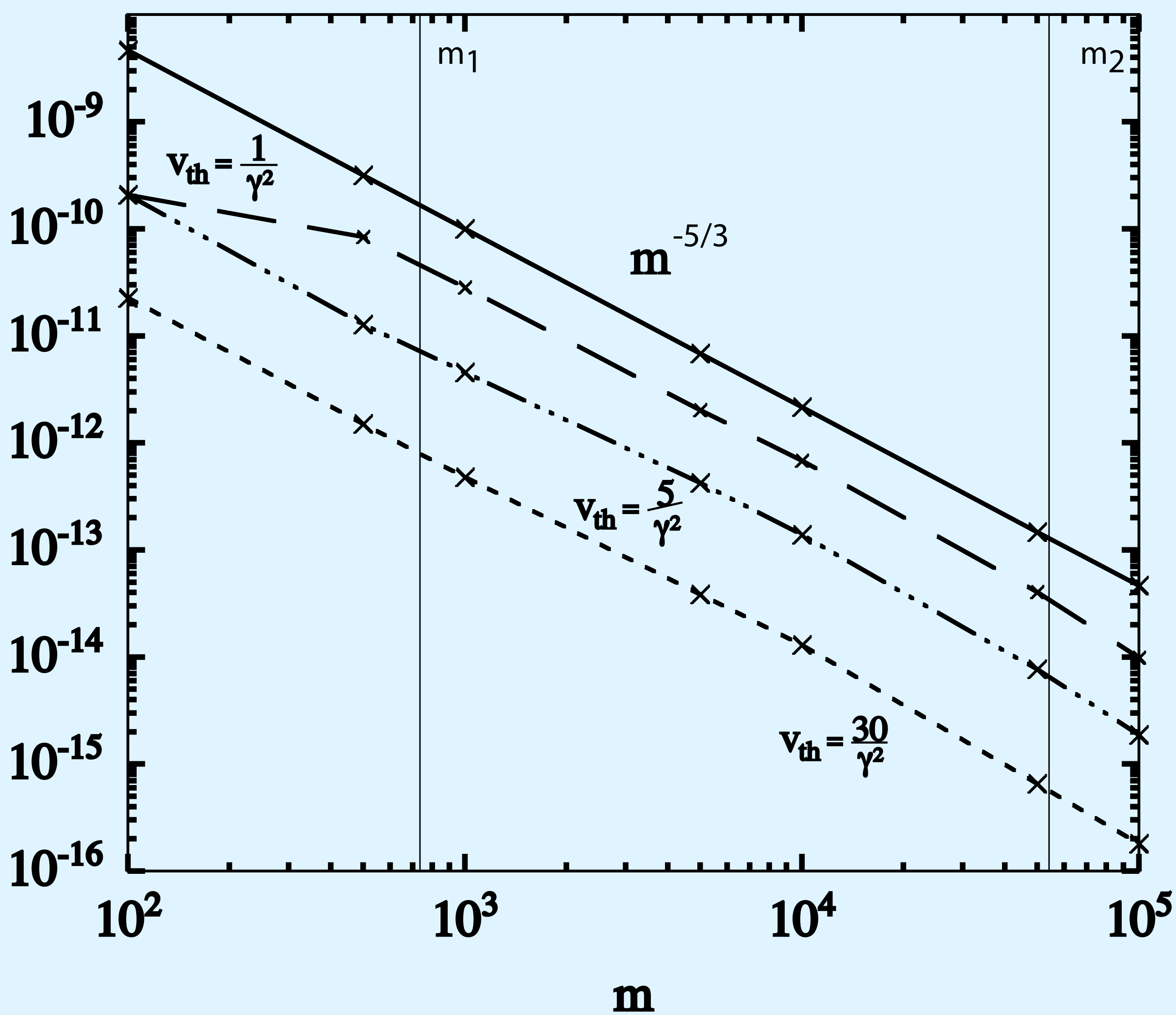
Together with the previously calculated growth rates and

$$\frac{dP_\phi}{dt} = -er\left[\delta E_\phi + (\mathbf{v}\times\delta\mathbf{B})_\phi\right]$$

the saturation amplitude can be calculated

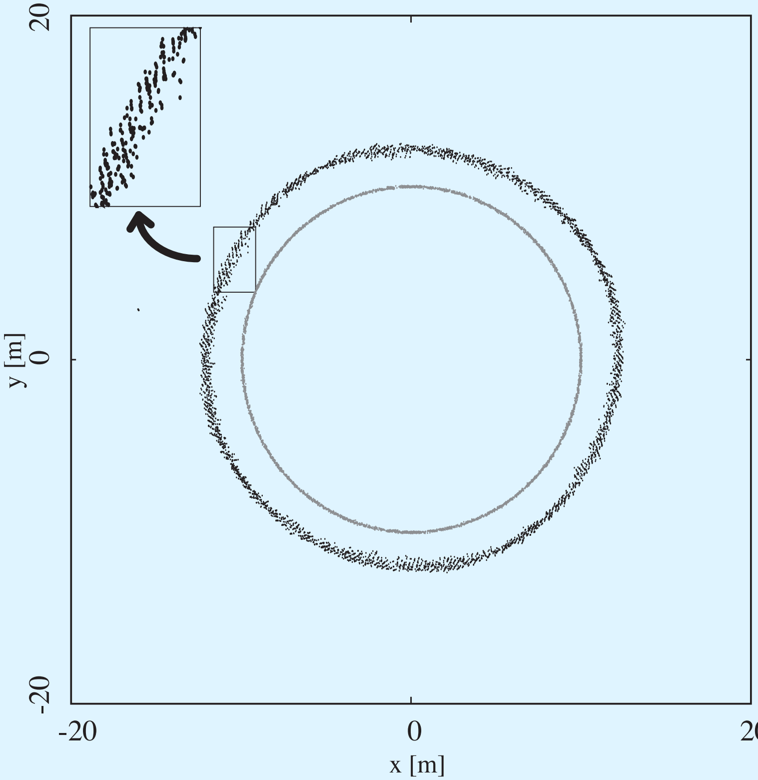
$$|\delta\Phi_{sat}|^2 = \frac{(m_e\gamma^3 v_\phi^2)^2}{e^2}\left(\frac{Im(\omega)}{m\dot{\phi}}\right)^4$$

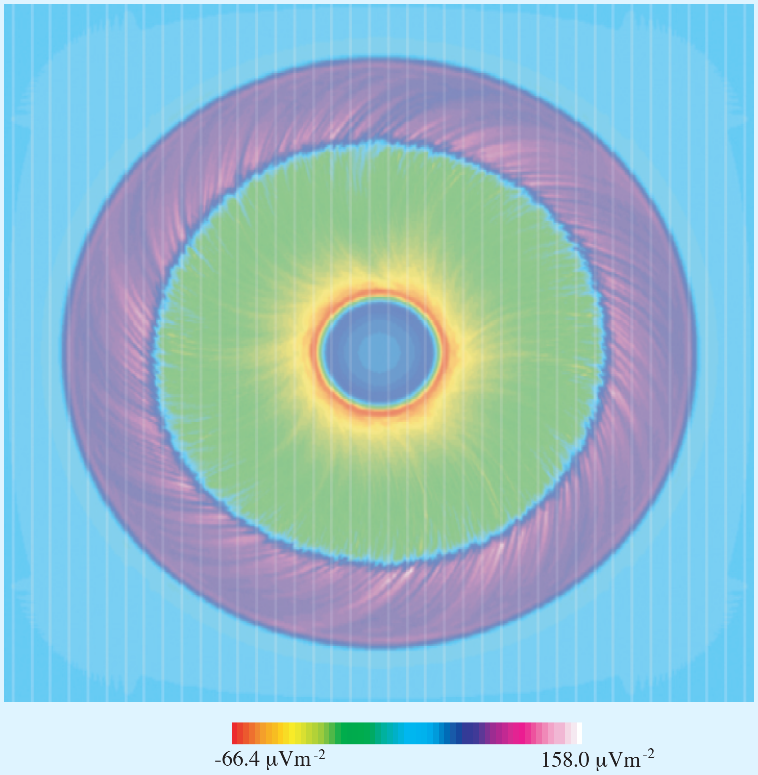
Brightness Temperatures of 10^{25} K can be achieved.



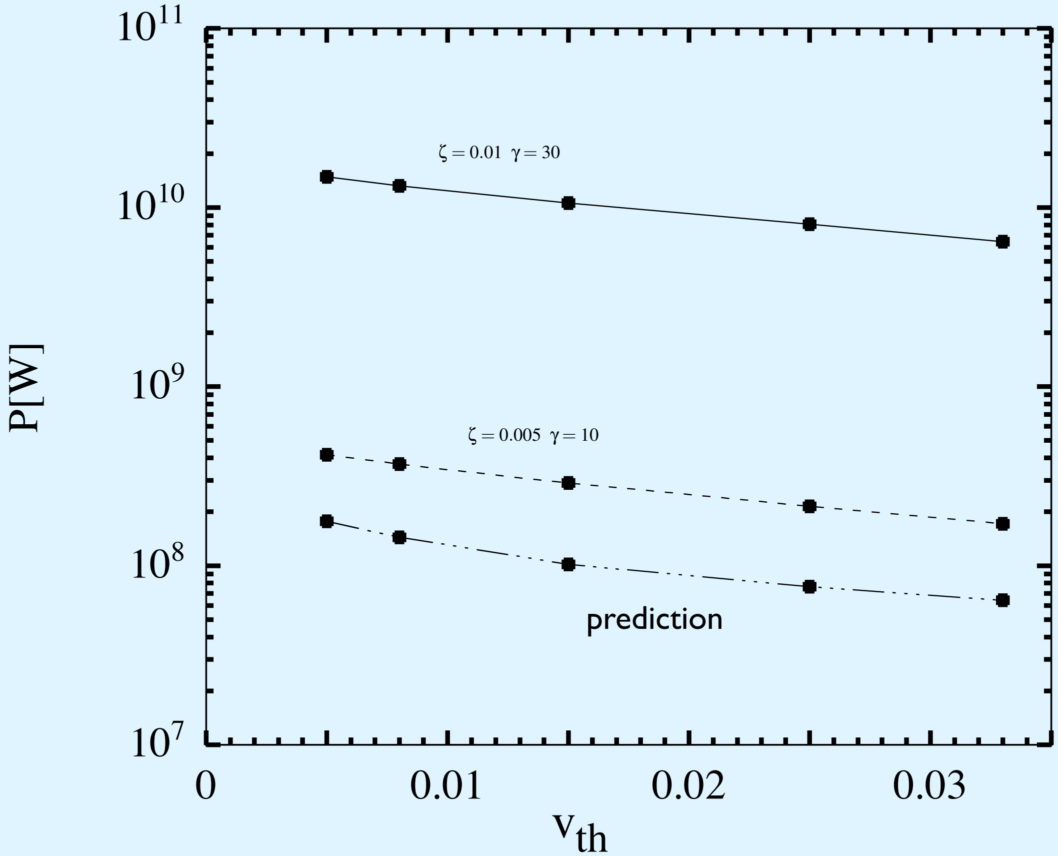
Power in arbitrary units

Particle-in-cell simulation





Total Power from OOPIC



MHD

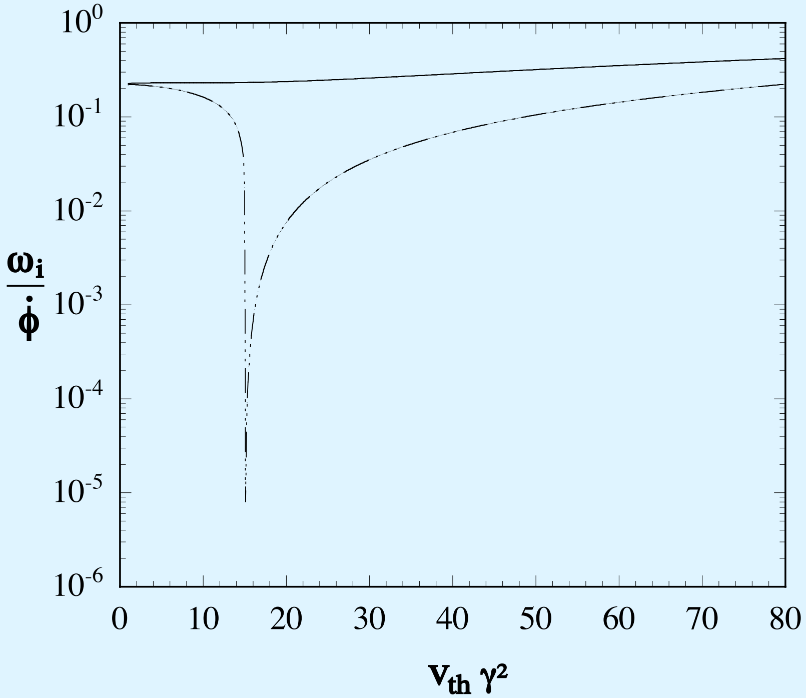
We solve the MHD equations for a Brillioun flow

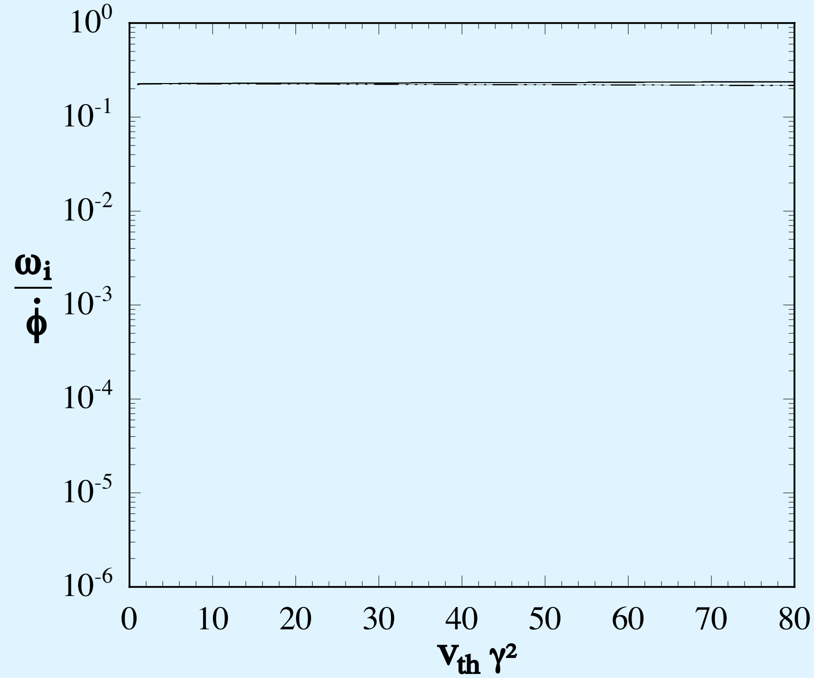
$$\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right](\gamma \mathbf{v}) = \frac{1}{m_e}\mathbf{F}$$

$$\frac{\partial}{\partial t}\rho + \nabla \cdot \mathbf{j} = 0$$

Similar results for large growth rates
Significant decoherence for small growth rates due to additional shear

Thicker layers





with Coulomb term

without Coulomb term